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DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

Massachusetts Institute of Technology Cambridge, Massachusetts 02139

> Final Report for Grant F49620–93-1-0604

A UNIFIED, MULTIRESOLUTION FRAMEWORK FOR AUTOMATIC TARGET DETECTION AND RECOGNITION

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I. Summary

In this final report we summarize our accomplishments in the research program supported by Grant AFOSR–93-1–0604 over the period from September 30, 1993 to July 31, 1997. The basic objective of this research program was to pursue an integrated set of research problems associated with Automatic Target Detection and Recognition (ATD/R). In particular, our research is "unified" in the sense that it involves the investigation of problems spanning the complete processing chain from sensor signal processing to image analysis to object recognition, thereby allowing us to understand how these individual processing stages interact and influence each other and to define new approaches that cut across established decompositions of the ATD/R problem. A second unifying theme of our work is the development of multiresolution methods. These range from the use of wavelet transforms and statistically optimal multiresolution algorithms that provide both extremely efficient procedures for image analysis and an explicit and rational basis for addressing noise/resolution tradeoffs to methods for multiresolution characterizations of object geometry and corresponding algorithms for extraction of multiresolution geometric features and for multiresolution object recognition.

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The principal investigators for this effort were Professors Alan S. Willsky, Jeffrey H. Shapiro, and W. Eric Grimson. Prof. P. Viola, and Drs. A.H. Krim, and W.M. Wells served as senior investigators for this project, and a considerable number of graduate research assistants and additional thesis students not requiring stipend or tuition support participated in this effort. Furthermore, in the course of our research, we established working relationships with several organizations with long histories of involvement in ATD/R. In particular, we have interacted closely with several groups of engineers and scientists at Lincoln Laboratory, including researchers directly involved in ARPA-sponsored ATD/R programs. We have also interacted with engineers at ERIM and have several collaborations with Alphatech, Inc., which is involved in several ATD/R projects under ARPA support, including a Phase II SBIR program, a project under the joint Industry-University initiative in ATD/R, and two contracts in the MSTAR program. Finally, we have been collaborating with TASC, again with regard to the MSTAR program.

At the end of this report we have included a complete listing of papers and reports that describe the work that has been performed under this project. As this substantial list suggests, we believe that we have had considerable success in our research. Moreover, our work has also led to several significant transitions which have helped to establish and fuel the industrial interactions mentioned previously. In addition, a major element of the

success that we have had can be seen in the continuing vitality of our efforts in this area. Indeed both our university research and our interactions with Lincoln, Alphatech, and ERIM have allowed us to identify research directions and areas for collaboration and transition that hold considerable promise for the some time to come.

In the body of this report we provide a brief summary of the major elements of our research program. Many of these have been highlighted in greater detail in earlier progress reports and are described in complete detail in the references included at the end of this report.

II. Research Description

In this section we briefly describe the major elements of the research in which we have been engaged during the tenure of this grant. We limit ourselves here to a succinct summary of these results and refer to the publications listed at the end of this report for detailed developments.

Multiresolution methods in laser range imaging

This work, which was led by Prof. Shapiro, addresses laser radar range imaging using realistic models for the uncertainty in the measurements provided by the sensor. Special attention is paid to the so-called range anomalies, which are due to nonlinear combined effect of laser speckle and receiver noise. We use a Haar-wavelet, expectation-maximization (EM) algorithm framework to produce a maximum-likelihood (ML) multiresolution approximation of the range truth. This allows us to tradeoff between the resolution of the reconstruction and the distortions caused by anomalies. That is, as we attempt to resolve finer and finer scale detail we are forced to use a greater percentage of the measured pixels and therefore are constrained in the number that can be declared to be anomalous. Using sensor statistics we can then determine the correct balance between resolution and anomaly rejection.

Through the work of Mr. Donald Greer, our Haar-wavelet range imaging been brought to a high state of development. By exploiting the block-nature of the Haar-wavelet multiresolution estimation procedure his Haar-wavelet estimation algorithm for maximum-likelihood fitting of P Haar-wavelet blocks to a Q-pixel image has computational complexity that is proportional to Q and independent of P. On a Sun SPARCstation 10, a 128 x 128 pixel laser radar data can process a 2 x 2 pixel resolution fit in 5 seconds. Moreover, this approach is fully parallelizable -- each Q/P-pixel block can be processed in independently -- placing true real time operation (60 frames/sec if desired on 128 x 128 pixel data) within reach. Most importantly, Mr. Greer's algorithm has outstanding numerical robustness, i.e., by avoiding matrix inversions it never encounters numerical problems associated with underflows in the EM algorithm's weights.

Statistically, Mr. Greer's EM/ML processor has proven to be essentially optimal -its estimation performance approaches the ultimate limit set by the complete-data form of
the Cramir-Rao bound -- with quantifiable performance characteristics. Mr. Greer's
software, a user manual, a full technical paper, and a sample input file plus processed
imagery are available through the worldwide web.

In follow-on research, Ms. Asuman Koksal has recently begun a master's thesis to wed Mr. Greer's fast EM/ML algorithm to a model-based object recognition algorithm developed by Dr. William Wells III of MIT. The combined system promises to be the first quasi-optimum end-to-end object recognition system for laser radar range imagery in which the analytical audit trail, i.e., the quantification of performance at every stage from anomaly suppression to recognition decision, is fully understood. Ms. Koksal's work benefits from a major release of DARPA-Lincoln Laboratory laser radar data. This data release includes a tutorial description of the MIT Lincoln Laboratory Infrared Airborne Radar (IRAR) sensor suite and a simple database cross-referencing the data files to their contents, as well as more than 500 Mbytes of actual sensor data files. IRAR, which was flown aboard a Gulfstream G-1 aircraft, was used to collect multispectral ground image data using active (laser radar) subsystems operating in at long-wave and near-visible infrared wavelengths, passive optical subsystems operating in the long-wave infrared band, and a millimeter-wave radar.

In addition, very recently Mr. Andrew Tsai, together with Prof. Willsky, have begun to investigate the integration of Prof. Shapiro's physics-based model for the errors in laser range measurements directly with prior statistical models for the scene being imaged in order to provide enhanced imaging based on Bayesian rather than likelihood estimation. This represents a first attempt to bring prior knowledge back into the earliest stages of image formation. A key here is the need to develop methods that are computationally efficient, a challenge for images of even moderate size when using many standard frameworks for statistical modeling of prior information. However, the multiresolution statistical modeling framework, described later in this report, appears to provide an ideal framework for accomplishing the goal of blending optimal estimation with the nonlinear EM algorithm required to deal with anomalies.

Multiresolution Models for Aspect-Dependent SAR Imaging

This effort, led by Prof. Shapiro, leverages off the work of Prof. Willsky and his collaborators (notably Dr. Chaney and Dr. Irving) which has used Lincoln Laboratory SAR data to demonstrate the efficacy of multiresolution discriminants for distinguishing targets from clutter as well as the benefits of exploiting the aspect-dependence (broadside flash) seen in foliage penetrating SAR imagery. Unlike these efforts, however, the new work builds from fundamental phenomenology originally established, by Prof. Shapiro and Dr. Dongwook Park, for the study of optical SAR. Initial results, being developed by Mr. Gilbert Leung in his M.Eng. thesis, appear to be quite promising.

Mr. Leung has compared the carrier-to-noise ratio (CNR) signatures produced in an idealized 1-D, i.e., continuous-wave, SAR by isolated specular and diffuse reflectors of the

same size. Whereas both reflectors produce the same average intensity image, in stripmap operation, when processed to full resolution, such is not the case for multiresolution processing. Specifically, the coherence of the specular reflector imparts a higher resolution capability in the intermediate processing regime wherein the along-track chirp compression filter has a processing time that is long enough to produce an effective synthetic aperture, but short enough that the maximum, i.e., transmitter-dwell-limited, synthetic aperture resolution has not been realized. He has shown that this multiresolution signature provides a powerful discriminant for separating man-made (specular) returns from natural terrain (diffuse) returns. This initial model establishes firm theoretical underpinning for the aspect-dependence (broadside flash) effects expected from a specular target9s tilt with respect to the radar-to-ground axis.

In very recently initiated follow-on work Mr. Chen-Pang Yeang begun to examine the extension of Mr. Leung's formalism in a variety of important ways. First, Mr. Yeang has established a model for fully polarimetric SAR operation. This model suggests, among other things, how to exploit the strong polarization signature of a dihedral reflector. Mr. Yeang has also generalized Mr. Leung's model to full 2D stripmap operation. In future work, Mr. Yeang will address both detection issues from a fundamental viewpoint and via examination of real data from the recently released MSTAR data set.

Multiresolution methods in image understanding

Our work in this area has had several components that build on other projects in which we have been involved as well as on some new ideas that involve the fusion of several different perspectives. In particular:

(a) Prof. Willsky's former student, Dr. Seema Jaggi, has recently completed her research on robust multiresolution feature extraction through a technique which we refer to as high-resolution pursuit. This work, has been carried out in direct collaboration with Prof. Stephane Mallat of Courant Institute,. The basic approach is a variation on Mallat's matching pursuit algorithm involving a new criterion that trades off between global fit and local fit in choosing each of a succession of features. This involves only a modest increase in complexity over matching pursuit but leads to features which are much more clearly connected to physical features in data and which appear to have significant robustness to several types of noise, including additive noise as well as "spiky" noise, as one would expect in speckle-corrupted radar data.

The initial application of this method has been to the problem of recognition of objects from their silhouettes. Specifically, there is a natural way in which to map a silhouette into a 1-D signal corresponding to the outline of the object, and a number of

recognition methods have been developed that are based on analyzing these 1-D silhouette functions. These methods all have some deficiencies, notably robustness to noise or to the absence of features or the introduction of spurious features due to noise, occlusion, or anomalous estimation of object silhouette as can occur in practice. The vehicle that we are using to test our approach is the recognition of silhouettes of a number of different aircraft, a problem that has been considered by others and which thus provides us with a fair way in which to compare our methods to those developed by others. The results of extensive testing demonstrates the superiority of our new approach, including an increased level of robustness to the types of silhouette errors observed in practice.

This technique has been transitioned to Alphatech, Inc., where high-resolution pursuit is being extended and applied to the problem of robust compression of SAR target models for template-based ATR and to problems of multisensor fusion. In addition, Dr. Hamid Krim of MIT and his student, Mr. Dewey Tucker, have recently developed a robust wavelet-based technique for optimal adaptive wavelet representation for noisy data and have begun an examination of the applicability of this method to high-resolution radar feature extraction. This application is being explored in conjunction with Dr. Ronald Chaney and Dr. Alan Chao of Alphatech.

(b) Dr. W. Wells, Dr. P. Viola and Prof. E. Grimson have developed a new approach both for finding the pose of an object in an image and for registering pairs of images. In earlier work we examined methods that utilized the Expectation/Maximization algorithm to trade off solving for the correspondence between model and data features, and solving for the pose of the target in the data coordinate frame. A multiresolution version of this method was designed and implemented to demonstrate the potential efficiencies and robustness gained by using multiresolution models and data. A drawback to this approach is a reliance on explicit uncertainty models. As an alternative, we have developed a new approach for finding the pose of an object model in an image and for registering pairs of images based on a new formulation of the mutual information between model and image or between pairs of images. As applied here, the technique is intensity-based rather than feature-based. It works well in domains in which edge or gradient-based methods have difficulty and is much more robust than traditional intensity-based methods such as those based on correlation. Indeed, the approach can be applied to data of very different modalities, in which correlation is inappropriate.

Our approach to alignment is based on the following steps: (1) the mutual information of the model and image, or the pair of images, is defined and expressed in terms of the entropies of several random variables; (2) the entropies and their derivatives

with respect to alignment parameters are approximated by stochastic approximation methods based on sampling the pair of images; and (3) a local maximum of the mutual information is sought by using stochastic approximation algorithms.

We have demonstrated the power of this method in several domains, including tracking moving 3D objects in video sequences and registering SAR with video or other data sources and registering multiple SAR data sets or target models with SAR data. The SAR applications have been explored, in part, in collaboration with Dr. Alan Chao at Alphatech, and our basic algorithms have now been transitioned to Alphatech.

Modeling, analysis, and processing of multiresolution stochastic models

This portion of our research has as its objective exploiting and extending our previous work on developing a methodology for the modeling of spatially-distributed data using multiresolution stochastic models. Under our ARPA project we have made recent progress in the following directions:

(a) In previous technical reports we described the work of Prof. Willsky and his students in developing a significant extension of the use of our scale-recursive multiresolution models for the modeling of spatial phenomena. These models are pyramidal in nature, and their local Markovian structure, both in scale and space, leads to extremely fast algorithms for efficient estimation, statistical inference, and sensor fusion. As we have reported previously, a version of the code for these algorithms is now available. In some of our more recent work, we have extended our framework to allow for nonlocal measurements--i.e., to allow for measurements at differing resolutions. These measurements show up as local measurements in our framework, but at different resolutions in our multiscale representation. As a result, our very efficient methods for optimal data assimilation allows us to fuse these different data sets with the same efficiency as for the processing of a single data set. In addition this work has led to a theory for the multiresolution modeling of fractal, 1/f processes. In particular, we have now shown that the variables captured by our models for such processes obey some very simple selfsimilarity and shift-invariance properties that makes the construction of such models extremely simple. This work has also spurred additional work that has recently commenced on a new, first principles approach to the multiresolution modeling of large, nonstationary fields in general and so-called multifractal processes that have fractal characteristics that may vary with both space and scale.

(b) In an early part of our research under this grant Prof. Willsky, his former student, Dr. William Irving (now at Alphatech), and Dr. Les Novak of Lincoln had considerable success in constructing and using multiresolution models for SAR imagery as the basis for enhancing Lincoln's discrimination algorithm, resulting in a reduction in false alarm rates by a factor of approximately 6 over Lincoln's baseline algorithm. Following this, Prof. Willsky, his student, Mr. Charles Fosgate, Dr. Hamid Krim of MIT, and Dr. Ronald Chaney (formerly of Lincoln Lab and currently at Alphatech) had considerable success in extending these multiresolution SAR likelihood ratio methods to the problem of distinguishing different types of natural terrain (trees, grass, etc.). The method we have developed has been shown to provide highly reliable classification decisions and accurate estimates of terrain boundaries. This work has very recently been documented in a paper appearing in the special issue of the IEEE Trans. on Image Processing on ATR. In addition, Dr. Krim and his student, Mr. Andrew Kim, have refined the methods developed by Fosgate and have extended them in order to develop efficient SAR compression algorithms that take advantage of the speckle-decorrelating property of our multiresolution models. This work will be presented at the Orlando SPIE meeting in April 1997. These methods have been transitioned both to Lincoln and recently to Alphatech, Inc.

In addition to these efforts, we have also recently initiated another new project aimed at exploiting the properties of our multiresolution SAR models even further. Specifically, Prof. Willsky, his new student, Mr. John Richards, and Dr. Irving have begun to look at the use of multiresolution SAR models for optimal or near-optimal model-based ATR. Specifically, since our models do a surprisingly good job of whitening speckle, they can provide the basis for optimal extraction of features corresponding to statistically significant deviations from this white behavior, which can be directly related to the presence of one or several dominant scatterers. In our work we expect to investigate both enhanced template-based methods as well as fully model-based methods, for which the problems of pose estimation, search, and match take on new twists, thanks to the multiresolution nature of our features. In addition, as an alternative direction we are also exploring the use of multiresolution models to capture aspect-dependent effects of significant scatterers. The key idea here is that the effect of pose in such a situation becomes more complex, since pose can effect not only the location but the appearance of aspect-dependent scatterers.

(c) In another portion of our research, Mr. Austin Frakt, a student of Prof. Willsky's developed a scale-recursive anomaly detection and localization methods from tomographic data. The objective of this project was to devise a decision-directed

mechanism to decide in which areas we should "zoom" in to resolve anomalies in more detail. What we have obtained is a result that we believe is significant well beyond its use in tomography. In particular, in trying to perform zooming, one might imagine forming a low-resolution image and performing detection on it--i.e., using the low resolution pixels as the statistics on which to base detections that would guide subsequent zooming operations. By interpreting the problem as one of composite hypothesis testing, it is not difficult to show that that approach is not optimal--indeed in statistical jargon, there is no uniformly most powerful test. However, it is possible to come up with other statistics that do a better job than that done by forming a low-resolution image. That is, what one does at coarser resolutions is to image statistics that are useful for zooming and the statistics that one images may not correspond to lower-resolution images of the phenomenon of interest. What this has led us to consider is the development of statistics that are optimized to maximally separate groups of hypotheses. We developed several alternative optimization criteria for this in the context of tomography and have finished an investigation of one of these that demonstrates the enhanced performance obtained using these optimized statistics over using coarse-scale imagery formed using conventional methods of image formation. Mr. Frakt is currently continuing to test this method and work on producing a paper documenting it.

(d) Prof. Willsky and his student Mr. Cedric Logan have recently developed the foundation of an estimation-theoretic approach to optimal SAR image formation when there is motion in the scene being imaged. The starting point for this work involves viewing the problem as one of joint position-velocity imaging--i.e., the SAR equivalent of range-Doppler imaging but now in 4 dimensions (2 space and 2 spatial velocity). This work is being carried out in collaboration with Drs. Krim and Chaney. Mr. Logan has developed a novel and very efficient method for calculating what can alternatively be thought of as the likelihood function for the location in range-rate/cross range of a scatterer or as an image in this 2-D space for each range bin. Using this likelihood function he has now developed a first approach to SAR imaging that does not require motion to be rigid body. Specifically, one of the objectives of this work is to develop a method that can focus an entireSAR image even if different scatterers in the scene are moving differently (including having a target moving across a stationary background). We have recently obtained promising results on simulated data demonstrating that the method can indeed focus images with non-rigid motion. We are also in the process of characterizing the blur in the higher-dimensional imaging framework in order to developed enhanced methods when there are interfering scatterers (e.g. due to background clutter and moving targets). One challenge here is the

development of a test environment that is both realistic and that allows in essence Monte Carlo testing in order to obtain statistically significant figures of merit for our approach. The idea here is not to use X-Patch or other simulators but instead to take real SAR data (in particular Lincoln ADTS data) and selectively modify phase histories in order to mimic the effects of motion. We have in mind developing a test capability that will be of value for many of our research efforts, including the work on multiresolution SAR-based ATR mentioned previously. This capability is being developed in collaboration with Drs. Chaney, Chao, and Irving as well as Dr. John Wissinger of Alphatech.

- (e) Prof. Willsky and another student, Mr. Michael Schneider, have developed efficient and statistically based multiresolution approaches to the segmentation of signals and images. The work on this to date is summarized in Mr. Schneider's Master's thesis. Major advantages afforded by our approach are the speed of our method and the direct availability of error statistics that are provided by our methodology (and that are essentially impossible to compute for standard variational formulations). The data on which we have focused for the most part in this work is surface height measurements (e.g. as one would get from IFSAR) and IR imagery, and in this context what our results show are not only excellent reconstructions but also the ability to accurately pick out objects from background (when there is a height or IR contrast differential) even in the presence of significant levels of noise.
- (f) Prof. Willsky, his student Mr. Ilya Pollak, and Dr. Krim have had a significant success in a new effort, aimed at developing robust image segmentation algorithms. Our approach is based on a detailed examination of the very active field of nonlinear diffusions and curve evolution in image processing, from which we developed a very simple variant that not only leads to vastly reduced computational burdens but also provides explicit segmentations at a hierarchy of resolutions (rather than requiring substantial postprocessing and interpretation). This new algorithm is characterized by a set of coupled differential equations for the transformation of image pixel values (analogous to the linear differential equations that lead to the scale-space concepts developed by Witkin and others), where in our case the differential equations have a signicant discontinuity that leads both to robust edge identification and to noise removal, including the removal of occasional high-amplitude noise spikes. We have recently demonstrated that this algorithm can perform surprisingly accurate segmentations for the very high speckle levels present in single-polarization SAR imagery. Given this success, we are currently investigating the extension and application of this methodology to problems such as robust feature extraction.

Publications

The publications listed below represent papers, reports, and theses supported in whole or in part under Grant AFOSR-93-1-0604:

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